SMALL COVER APPROACH TO THE SUPREMA OF POSITIVE CANONICAL PROCESSES

(PARALLEL TALK)

WITOLD BEDNORZ

The aim of this talk is to show a new idea in bounding positive canonical processes based on random variables that satisfy a minimal tail decay assumption. This is a joint work by Rafal Martynek, Rafal Meller and me. The problem concerns equiavlent characterization of $S(T) = \mathbb{E}\sup_{t\in T}\sum_{i=1}^{d} t_i X_i$, where X_i are independent positive random variables and $t_i \geq 0$. The approach is based on the existence of a small covering, which was conjectured by Michel Talagrand to describe $S(T)$ in the case of selectors, i.e. when X_i are i.i.d and $X_i \in \{0,1\}$. The conjecture was proved by Park and Pham also a much simplified prove was given by our team.

QUASI-INFINITELY DIVISIBLE DISTRIBUTIONS
(PARALLEL TALK)

DAVID BERGER, TU DRESDEN (ALEXANDER LINDNER, MERVE KUTLU)

A probability measure μ on $\mathbb{R}^n$ is called infinitely divisible if for all $n \in \mathbb{N}$ there exists a probability measure μ_n such that $\mu_n^{*n} = \mu$. Infinitely divisible distributions are a well-studied and well-understood class of probability measures and the notion of Lévy processes is closely related to the infinite divisibility property. In contrast a quasi-infinitely divisible distribution μ is defined via a factorization problem: We call μ quasi-infinitely divisible (QID) if there exists an infinitely divisible measure μ_1 such that $\mu * \mu_1$ is again infinitely divisible. In this talk we discuss differences between quasi-infinite divisibility and infinite divisibility and show that invertible probability measures with respect to the convolution are QID.

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LARGE SOLUTIONS FOR SUBORDINATE SPECTRAL LAPLACIAN
(PARALLEL TALK)

IVAN BIOČIĆ, UNIVERSITY OF TURIN (JOINT WORK WITH VANJA WAGNER)

Let $D \subset \mathbb{R}^d$, $d \geq 3$, be a $C^{1,1}$ bounded domain and $f : \mathbb{R} \to [0, \infty)$. In this talk, we find a solution to the following semilinear problem

$$\begin{array}{rcll} \phi(-\Delta|_D)u & = & -f(u) & \text{in } D, \\ \frac{u}{P_D^\phi \sigma} & = & \infty & \text{on } \partial D. \end{array} \tag{1}$$

Here the operator $\phi(-\Delta|_D)$ is an extension of the infinitesimal generator of a subordinate killed Brownian motion, where the subordinator has ϕ as its Laplace exponent, and $P_D^\phi \sigma$ is a reference function defined as the Poisson potential of the $d-1$ dimensional Hausdorff measure σ on ∂D for the subordinate killed Brownian motion. The solution to (1) is called a large solution since it cannot be uniformly bounded by a nonnegative $\phi(-\Delta|_D)$-harmonic function. This setting covers and extends the case of the spectral fractional Laplacian since we assume that ϕ satisfies the weak scaling condition at infinity with powers δ_1 and δ_2.

We also prove higher regularity results for $\phi(-\Delta|_D)$ in classical Hölder spaces.

Theorem 1. *Let $f \in C^{k+\alpha}(D)$ and let $u \in L^1(D, \delta_D(x)dx)$ solve*

$$\phi(-\Delta|_D)u = f \quad \text{in } D$$

in the distributional sense. Then $u \in C^{k+\alpha+2\delta_1}(D)$. Furthermore, for any given sets $K \subset\subset K' \subset\subset D$, there exists a constant $C = C(d, \alpha, K, K', D, \phi) > 0$ such that

$$||u||_{C^{k+\alpha+2\delta_1}(K)} \leq C \left(||f||_{C^{k+\alpha}(K')} + ||u||_{L^1(D,\delta_D(x)dx)} \right).$$

Moreover, if $f = 0$, i.e. u is $\phi(-\Delta|_D)$-harmonic, then $u \in C^\infty(D)$ and the equality $\phi(-\Delta|_D)u(x) = 0$ holds at every point $x \in D$.

BEURLING–DENY FORMULA FOR SOBOLEV–BREGMAN FORMS
(PARALLEL TALK)

MICHAŁ GUTOWSKI, WROCŁAW UNIVERSITY OF SCIENCE AND TECHNOLOGY

One way to investigate symmetric Markov processes involves the theory of Dirichlet forms, which corresponds with L^2 space. It is well known that there is a one-to-one correspondence between the class of regular Dirichlet forms $\mathcal{E}$ and the class of symmetric Hunt processes (strong Markovian, quasi-left continuous with càdlàg paths).

The Beurling–Deny formula provides the following unique decomposition of a regular Dirichlet form $\mathcal{E}$, defined on an appropriate domain $\mathcal{D}(\mathcal{E})$ in L^2 into an (inexplicit) strongly local part, the jumping part, and the killing part:

$$\mathcal{E}(u,v) = \mathcal{E}^{(c)}(u,v) + \frac{1}{2} \iint\limits_{(E\times E)\setminus \mathrm{diag}} (u(y)-u(x))(v(y)-v(x))J(dx,dy) + \int\limits_E u(x)v(x)k(dx),$$

where J is the *jumping measure*, a k is the *killing measure*. Here u, v are continuous (or, more generally, quasi-continuous versions of) functions from the domain $\mathcal{D}(\mathcal{E})$. The three parts describe the local, jumping and killing behavior of the corresponding Hunt process.

The Sobolev–Bregman form $\mathcal{E}_p$ is an extension to L^p of the Dirichlet form $\mathcal{E}$. The applications of this notion can be found in [2, 3, 4, 5, 6, 7], although the name *Sobolev–Bregman* was introduced only recently in [5].

Our goal is to derive the following Beurling–Deny formula for $\mathcal{E}_p$:

$$\mathcal{E}_p[u] = \frac{4(p-1)}{p^2}\mathcal{E}^{(c)}(u^{\langle p/2\rangle}, u^{\langle p/2\rangle}) + \frac{1}{p} \iint\limits_{(E\times E)\setminus \mathrm{diag}} F_p(u(x),u(y))J(dx,dy) + \int\limits_E |u(x)|^p k(dx),$$

where the function $F_p(a,b) := |b|^p - |a|^p - pa^{\langle p-1\rangle}(b-a)$ is so-called *Bregman divergence* and $a^{\langle\kappa\rangle} := |a|^\kappa \operatorname{sgn} a$.

The talk is based on the joint work [1] with Mateusz Kwaśnicki (Wrocław University of Science and Technology).

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PICK-NEVANLINNA FUNCTIONS AND INVERSE TIMES OF SPECTRALLY NEGATIVE LÉVY PROCESSES.

(PARALLEL TALK)

WISSEM JEDIDI, KING SAUD UNIVERSITY AND FACULTY OF SCIENCE OF TUNIS (JOINT WORK WITH ZBIGNIEW JUREK, AND NUHA AL TAYMANI)

The following question is due to several discussions of WJ with Loïc Chaumont. *Let ϕ be the Bernstein function of some subordinator ξ, such that it is impossible to check with standard calculations that ϕ^{-1} the Laplace exponent of spectrally negative Lévy process (SNLP). Is there any method indicating that ξ is the first passage time of a SNLP?*

To answer the latter, we show a property of temporal complete monotonicity, similar to the one obtained via the Lamperti transformation by Bertoin & Yor for self-similar Markov processes (*On subordinators, self-similar Markov processes and some factorizations of the exponential variable*, Elect. Comm. in Probab., vol. 6, pp. 95–106, 2001). More precisely, we show the remarkable fact that for a subordinator ξ, the function $t \mapsto t^n\,\mathbb{E}[\xi_t^{-p}]$ is, depending on the values of the powers $n = 0, 1, 2,\ p > -1$, or a Bernstein function or a completely monotone function. In particular, ξ is the inverse time subordinator of a SNLP, if, and only if, for some $p \geq 1$,

$$t \mapsto t\,\mathbb{E}[\xi_t^{-p}] \text{ is a Stieltjes transform.}$$

We provide several explicit examples for the latter. We will also clarify to what extent Nevanlinna-Pick functions are related to free-probability and to Voiculescu transforms, and we provide an inversion procedure. Finally, we establish new connections between the class of Nevanlinna-Pick functions and that of Lévy processes.

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QUASI-ERGODICITY OF COMPACT STRONG FELLER SEMIGROUPS ON L^2

(PARALLEL TALK)

KAMIL KALETA, WROCŁAW UNIVERSITY OF SCIENCE AND TECHNOLOGY

I will discuss some of the results from the preprint [1], written jointly with R.L. Schilling (TU Dresden), concerning the quasi-ergodic properties of strong Feller semigroups of compact integral operators $\{U_t : t \geq 0\}$ on $L^2(M, \mu)$, where M is a locally compact Polish space equipped with a locally finite Borel measure μ. We assume that the considered semigroups (and their dual semigroups) are ultracontractive, and their operators improve positivity, but we do not require self-adjointness or even normality.

The first result relates exponential quasi-ergodicity of the considered semigroups on $L^p(M, \mu)$ and the uniqueness of the quasi-invariant measure with finiteness of the heat content (for large times). This estimate is quite practical, but it does not always provide a good control in the space. The second result provides a complete characterization of a slightly stronger property, where convergence is uniform with respect to a certain increasing family of sets (indexed by time) filling the space, and the rate of convergence depends on the decay rate of the function $U_{t_0}\mathbf{1}_M(x)$ (for some $t_0 > 0$) at infinity. Both results heavily depend on ground states.

One of the motivations for these investigations was to obtain a complete description of the quasi-ergodicity properties of evolution semigroups associated with a certain class of non-local Schrödinger operators with confining potentials, for which sharp estimates of ground states [2] and integral kernels [3] are known.

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HARDY AND RELLICH INEQUALITIES FOR ANTISYMMETRIC AND ODD FUNCTIONS

(PARALLEL TALK)

MICHAŁ KIJACZKO (WROCŁAW UNIVERSITY OF SCIENCE AND TECHNOLOGY)

The classical Hardy inequality on $L^p(\mathbb{R}^d)$ states that

$$\int_{\mathbb{R}^d} |\nabla u(x)|^p \, dx \geq \left(\frac{|d-p|}{p} \right)^p \int_{\mathbb{R}^d} \frac{|u(x)|^p}{|x|^p} \, dx,$$

where $u \in W^{1,p}(\mathbb{R}^d)$, if $p < d$ and $u \in W^{1,p}(\mathbb{R}^d \setminus \{0\})$, if $p > d$. Recently [1, 2, 3, 4], there has been a research interest connected to the development of the Hardy inequality for antisymmetric and odd functions, but only in the case of $p = 2$. It turns out that the sharp constant in this case is bigger. In this talk we investigate Hardy and Rellich inequalities for classes of antisymmetric and odd functions and general exponent $p \geq 2$. The obtained constants are better than the classical ones.

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PARTICLE SYSTEMS AND CRITICAL STRENGTHS OF GENERAL SINGULAR INTERACTIONS

(PARALLEL TALK)

DAMIR KINZEBULATOV (UNIVERSITÉ LAVAL)

The talk discusses global weak and strong well-posedness and other properties of interacting partice system

$$dX_i = -\frac{1}{N}\sum_{j=1, j\neq i}^{N} K_{ij}(X_i - X_j)dt + \sqrt{2}dB_i, \quad i = 1, \dots, N, \tag{1}$$

where $[0,\infty[\ni t \mapsto X_i(t)$ is the trajectory of the i-th particle in $\mathbb{R}^d$, $\{B_i(t)\}_{t\geq 0}$ are d-dimensional independent Brownian motions, $d \geq 3$. The singular interaction kernels K_{ij} are form-bounded

$$\|K\varphi\|_2^2 \leq \kappa\|\nabla\varphi\|_2^2 + c_\kappa\|\varphi\|_2^2, \quad \varphi \in W^{1,2} \tag{2}$$

or satisfy similar conditions that allow to take into account the repulsion-attraction structure of K_{ij}. This includes K_{ij} in the weak L^d class or in a large Morrey class. Some concrete examples are

$$K(y) = \pm\sqrt{\kappa}\frac{d-2}{2}\frac{y}{|y|^2}, \quad y \in \mathbb{R}^d, \tag{3}$$

where the sign in front determines whether the interaction is attracting or repulsing. It is known that in the attracting case in (3), if κ is too large, then the attraction starts to dominate over the diffusion, and the particle system ceases to be well-posed. We are searching for the assumptions on κ in (2) that withstand the passage to the limit $N \to \infty$, aiming at the proof of the existence of the mean field limit. Our results, stated briefly, are as follows.

Theorem 1. *The following are true:*

(i) If $\kappa < 4(\frac{N}{N-1})^2$, then there exists a strong Markov family of martingale solutions of particle system (1). *Under additional assumptions on κ, one has conditional weak uniqueness and strong well-posedness for* (1).

(ii) For the model attracting interaction kernel

$$K(y) := \sqrt{\kappa}\frac{d-2}{2}\frac{y}{|y|^2}, \quad y \in \mathbb{R}^d,$$

the heat kernel of (1) *admits an explicit (necessarily non-Gaussian) upper heat kernel bound in the regions where the particles are close to each other.*

(iii) For the borderline strengths of interactions, the corresponding to (1) *Liouville equation on the torus is well-posed in a "critical" Orlicz space situated close to L^1.*

The proofs of (i) and (ii) use an L^p variant of De Giorgi's method, as is needed to "decouple" a tightness estimate from any strong gradient bounds, and an abstract desingularization theorem established earlier by Kinzebulatov-Semënov-Szczypkowski (2020).

BÔCHER TYPE THEOREM FOR GRADIENT PERTURBED LÉVY OPERATORS: SUPERCRITICAL AND SUBCRITICAL CASES
(PARALLEL TALK)

TOMASZ KLIMSIAK,
NICOLAUS COPERNICUS UNIVERSITY IN TORUŃ,
INSTITUTE OF MATHEMATICS PAS

Bôcher's classical theorem (see [1]) asserts that any positive harmonic (with respect to the Laplacian) function $h : B(0,1) \setminus \{0\} \to \mathbb{R}$ (here $B(0,1) := \{x \in \mathbb{R}^d : |x| < 1\}$ and $d \geq 2$) can be expressed as a linear combination of the Green function for the unit ball and a positive function that is harmonic in the whole unit ball. Nearly eighty years later, in the context of the theory of isolated singularities, H. Brezis and P.L. Lions generalized Bôcher's theorem (see [2]) to positive distributional solutions of

$$-\Delta u + f \geq 0 \quad \text{in} \quad B(0,1) \setminus \{0\},$$

where $f \in L^1_{\text{loc}}(B_1)$. Recently Congming Li et al. (see [3]) extended this result to positive distributional solutions of

$$-\Delta^{\alpha/2} u + b(\cdot)\nabla u + f \geq 0 \quad \text{in} \quad B(0,1) \setminus \{0\},$$

with $\alpha \in (1, 2]$ (subcritical case). The critical and supercritical cases have been left as open problems.

We solve positively the open problem and show even more that the result by Congming Li et al. holds true with the fractional Laplacian replaced by any Lévy operator A satisfying Hartman-Wintner condition:

$$\frac{\mathrm{Re}\psi(\xi)}{\log(1 + |\xi|)} \to \infty, \quad |\xi| \to \infty,$$

where ψ is the characteristic exponent (Fourier multiplier) of A.

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FAVOURITE SITES OF A RANDOM WALK IN SPARSE RANDOM ENVIRONMENT

(PARALLEL TALK)

ALICJA KOŁODZIEJSKA, UNIVERSITY OF WROCŁAW

A random walk in sparse random environment is a model in which a particle performs a simple random walk on $\mathbb{Z}$. The movement is symmetric apart from some randomly chosen sites, where we impose random drift. This model may be seen as being in-between a classical, simple symmetric random walk and a well-studied walk in i.i.d. random environment. Therefore we may expect that, depending on the distribution of the environment, it should manifest properties resembling one or the other. During the talk, I will show how this interplay is visible in the annealed limit theorems for maximal local time of the walk. I will present two cases, one resembling a walk in i.i.d. environment, and the other, in which the sparsity of the environment becomes significant. I will briefly discuss which properties of the environment lead to this change of regime.

A LAW OF THE ITERATED LOGARITHM FOR ITERATED RANDOM WALKS, WITH APPLICATION TO RANDOM RECURSIVE TREES

(PARALLEL TALK)

VALERIYA KOTELNIKOVA, TARAS SHENCHENKO NATIONAL UNIVERSITY OF KYIV

Consider a general branching process (a.k.a Crump-Mode-Jagers process) generated by an increasing random walk whose increments have finite second moment. Let $Y_k(t)$ be the number of individuals in generation $k \in \mathbb{N}$ born in the time interval $[0, t]$. I shall discuss a law of the iterated logarithm for $Y_k(t)$ with fixed k, as $t \to +\infty$. As a corollary, I shall also present a law of the iterated logarithm for the number of vertices at a fixed level k in a random recursive tree, as the number of vertices goes to ∞.

The talk is based on the joint article [1] with Alexander Iksanov (Kyiv) and Zakhar Kabluchko (Münster).

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THE STOCHASTIC EVOLUTION OF A GENERAL CONTACT MODEL: AN ANALYTIC APPROACH

(PARALLEL TALK)

JURIJ KOZICKI, MARIA CURIE-SKŁODOWSKA UNIVERSITY

States of large populations dwelling in a habitat X are modeled as Radon counting measures defined on X – typically a locally compact Polish space, cf. [1]. For Γ being the set of all such measures, $\gamma \in \Gamma$ and a compact $\Lambda \subset X$, $\gamma(\Lambda) \in \mathbb{N}_0$ is the number of population members contained in Λ. By employing probability measures $\mu \in \mathcal{P}(\Gamma)$ as mixed population states, one passes to the statistical description as well as gets the possibility to take into account random acts of the population evolution. The latter is described as maps $[0, +\infty) \ni t \mapsto \mu_t \in \mathcal{P}(\Gamma)$ which solve appropriate evolution equations. For modelers, an important characteristic is given by the *occupation probabilities* $p_n(t, \Lambda) = \mu_t(\Gamma_\Lambda^{(n)})$, $\Gamma_\Lambda^{(n)} := \{\gamma \in \Gamma : \gamma(\Lambda) = n\}$, or by the corresponding falling factorial moments $\varphi_t(n, \Lambda) = \phi_{t,\Lambda}^{(n)}(1)$ generated by $\phi_{t,\Lambda}(s) = \sum_{n\geq 0} s^n p_n(t, \Lambda)$, cf. [3]. Note that, for $p_n(t, \Lambda) = (1 - q_{t,\Lambda})q_{t,\Lambda}^n$, one has $\varphi_t(n, \Lambda) = n!q_{t,\Lambda}^n$. If the evolution is Markov, one employs the Fokker-Planck equation

$$\mu_t(F) = \mu_0(F) + \int_0^t \mu_s(LF)ds, \quad \mu(F) := \int_\Gamma F d\mu, \tag{1}$$

where $F : \Gamma \to \mathbb{R}$ is a test functiion taken from a sufficiently representative class $\mathcal{F}$, whereas L is the generator that takes into account the aforementioned random acts. In this case, one speaks of the solutions of (1) for $(L, \mathcal{F}, \mu_0)$. The model which we study corresponds to

$$(LF)(\gamma) = \int_X [F(\gamma + \delta_x) - F(\gamma)]\, \zeta_\gamma(dx) + \int_X [F(\gamma - \delta_x) - F(\gamma)]\, m(x)\gamma(dx), \tag{2}$$

where δ_x is Dirac's measure, $\zeta_\gamma(dx) = \int_X \kappa(y, dx)\gamma(dy)$ and κ is a positive Radon measure kernel. The first (resp. second) term in (2) describes the appearance (resp. disappearance) of the population members. The existing population attracts the incomers through the kernel κ. Such models are also interpreted as birth-and-death models [2], and the version as in (2) is the most general among those study so far.

Let $\mathcal{P}_*$ be the set of all $\mu \in \mathcal{P}(\Gamma)$ such that the falling factorial moments satisfy $\varphi_t(n, \Lambda) \leq n!q_{t,\Lambda}^n$ for appropriate $q_{t,\Lambda} \in (0, 1)$. The key problem of our study is whether the evolution related to (2) exists and preserves $\mathcal{P}_*$, wheras its most sensitive aspect is the choice of $\mathcal{F}$. The result is the statement that, for a tediously selected $\mathcal{F}$ and any $\mu_0 \in \mathcal{P}_*$, there exists a unique solution $t \mapsto \mu_t$ of (1) for $(L, \mathcal{F}, \mu_0)$ such that $\mu_t \in \mathcal{P}_*$ for all $t > 0$. Its proof is based upon functional analytic methods developed for this purpose.

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EXPANSIONS OF DIFFUSION HEAT KERNELS BASED ON THE PARAMETRIX METHOD

(PARALLEL TALK)

OLEKSII KULYK, WROCŁAW UNIVERSITY OF SCIENCE AND TECHNOLOGY

We will discuss recent developments in small time approximation of heat kernels of diffusions, based on both classical and non-classical parametrix technique. Such approximations have natural applications in simulation procedures and analysis of asymptotic properties of statistical models, which will be also discussed in the talk.

The talk is based on joint research with Arturo Kohatsu-Higa (Ristumeikan University, Japan), Dmytro Ivanenko (Taras Shevchenko National University of Kyiv) and Krsysztof Bogdan (Wrocław University of Science and Technology).

ASYMPTOTICS AND GEOMETRIC FLOWS FOR A CLASS OF NONLOCAL CURVATURES
(PARALLEL TALK)

JULIA LENCZEWSKA, WROCŁAW UNIVERSITY OF SCIENCE AND TECHNOLOGY

We consider a family of nonlocal curvatures determined through a kernel which is symmetric and bounded from above by a radial and radially non-increasing profile. It turns out that such definition encompasses various variants of nonlocal curvatures that have already appeared in the literature, including fractional curvature and anisotropic fractional curvature. Our main goal is to study the limit behaviour of the introduced nonlocal curvatures under an appropriate limiting procedure. This enables us to recover known asymptotic results e.g. for fractional curvature, but also for anisotropic fractional curvature where we identify the limit object as a curvature being the first variation of the related anisotropic perimeter.

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SELF-SIMILAR SOLUTION FOR FRACTIONAL LAPLACIAN IN CONES

(PARALLEL TALK)

ŁUKASZ LEŻAJ, WROCŁAW UNIVERSITY OF SCIENCE AND TECHNOLOGY

We construct a self-similar of the heat equation for the fractional Laplacian with Dirichlet boundary conditions in every fat cone. Furthermore, we give the entrance law from the vertex and the Yaglom limit for the corresponding killed isotropic stable Lévy process and precise large-time asymptotics for solutions of the Cauchy problem in the cone.

The talk is based on a joint work [1] with Krzysztof Bogdan, Piotr Knosalla and Dominika Pilaczyk.

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MOMENTS AND TAILS OF HITTING TIMES OF BESSEL PROCESSES AND CONVOLUTIONS OF ELEMENTARY MIXTURES OF EXPONENTIAL DISTRIBUTIONS

(PARALLEL TALK)

RAFAŁ M. ŁOCHOWSKI, WARSAW SCHOOL OF ECONOMICS (JOINT WORK WITH WITOLD M. BEDNORZ)

I will present explicit estimates of right and left tails and exact (up to universal, multiplicative constants) estimates of tails and moments of hitting times of Bessel processes. The latter estimates are obtained from more general estimates of moments and tails established for convolutions of elementary mixtures of exponential distributions.

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CLUSTER SIZES IN SUBCRITICAL SOFT BOOLEAN MODELS
(PARALLEL TALK)

LUKAS LÜCHTRATH, WIAS BERLIN (JOINT WORK WITH BENEDIKT JAHNEL AND MARCEL ORTGIESE)

In continuum percolation theory, spatially embedded random graphs are build as such: the vertex set is given by a standard Poisson point process in Euclidean space and each pair of vertices is connected by an edge with a probability that depends in some way on the vertices'. A key question of interest is then whether there exists a non-trivial *percolation phase transition.* That is, the existence of a non trivial critical intensity value such that the graph contains only finite components if the Poisson process has intensity smaller than this critical value but contains an infinite component a.s. for larger intensities.

Arguably the two most well-known models in this context are the random connection model, in which each pair of vertices forms an edge independently of all other edges with a probability decaying polynomially in the vertices' distance, and the Boolean model, in which each vertex is assigned a (heavy-tailed) radius and two vertices form an edge if their associated balls intersect. For both models, the existence of a percolation phase transition under mild integrability conditions is well established. Moreover, further results for the cardinality and the spatial spread out of typical finite clusters in subcritical regimes have been proved.

In this talk, we consider the *soft Boolean model* introduced in [1] as a model that interpolates nicely between the random connection and the Boolean model. That is, in addition to the 'Boolean' edges resulting from the overlap of the assigned balls, further edges are added based on a polynomially decaying connection probability involving both, the spatial distance and the radii of the potential end vertices. In the whole parameter regime where there is a subcritical percolation phase [2], we consider a typical cluster and study its *Euclidean diameter* (the maximum of all spatial distances between two vertices of that cluster) and the *number of vertices* contained in it. For the Euclidean diameter we derive two phases: one phase where the radii are not strong enough compared to the long edge from standard random connection model. In this phase, the Euclidean diameter remains qualitatively unchanged compared to the random connection model. In the other phase the combination of both effects leads to a much larger diameter. On the contrary, we further show that the number of points is mainly driven by the large radius vertices alone. Hence, the cardinality of the cluster remains qualitatively unchanged (in the sense of the decay of its tail probability function) compared to the standard Boolean model. Put differently, adding long-range edges from the random connection model to the Boolean model in a subcritical regime significantly increases the spatial spread out of a typical cluster without qualitatively increasing its cardinality. We describe from which effects both quantities are mainly driven and sketch the most important proof steps.

References

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COMPARISON FOR THE LIPSCHITZ MAPPINGS OF GAUSSIAN AND BERNOULLI PROCESSES
(PARALLEL TALK)

RAFAŁ MARTYNEK, UNIVERSITY OF WARSAW, JOINT WORK WITH WITOLD BEDNORZ

In the paper “A chain rule for the expected suprema of Gaussian processes” A. Maurer derived a comparison result which states that the expected supremum of a Gaussian process indexed by the image of an index set with respect to the class of 1-Lipschitz functions can be controlled by the expected supremum over the original index set and the supremum depending of the class function. The result follows from the Majorising Measure Theorem and the concentration of measure. I'll discuss a possible sharpening of this result by constructing an appropriate sequence of partitions and applying the chaining method. Analogous result can be stated also for Bernoulli processes.

NONLINEAR FOKKER–PLANCK EQUATIONS AS GRADIENT FLOWS ON THE SPACE OF PROBABILITY MEASURES

(PARALLEL TALK)

MARCO REHMEIER, SCUOLA NORMALE SUPERIORE PISA (JOINT WORK WITH MICHAEL RÖCKNER)

We propose a general method to identify nonlinear Fokker–Planck equations (FPEs) as gradient flows on the space of probability measures on $\mathbb{R}^d$ with a natural differential geometry. Our notion of gradient flow does not depend on any underlying metric structure such as the Wasserstein distance, but is derived from purely differential geometric principles. We explicitly identify the associated energy functions and show that these are Lyapunov functions for the FPE-solutions. These results cover classical and generalized porous media equations, where the latter have a generalized diffusivity function and a nonlinear transport-type first-order perturbation.

ON NONLOCAL NEUMANN PROBLEM AND CORRESPONDING STOCHASTIC PROCESS

(PARALLEL TALK)

PAWEŁ SZTONYK, WROCŁAW UNIVERSITY OF SCIENCE AND TECHNOLOGY

We consider the following *Neumann problem for the fractional Laplacian* introduced by Dipierro et al. in [1]:

$$\begin{cases} (-\Delta)^{\alpha/2} u & = & f, \quad \text{in } D, \\ \mathcal{N}_{\alpha/2} u & = & f, \quad \text{in } \mathbf{R} \setminus \overline{D}, \end{cases} \tag{1}$$

where $\mathcal{N}_{\alpha/2}$ denotes so-called *nonlocal normal derivative*, defined by

$$\mathcal{N}_{\alpha/2} u(x) := \int_D (u(x) - u(y)) \nu(x,y)\, dy, \qquad x \in \mathbf{R} \setminus \overline{D}, \tag{2}$$

$\nu(x,y) = c_{\alpha,d} |y-x|^{-1-\alpha}$, $D = (0,\infty)$, and $\alpha \in (0,2)$. Following the probabilistic interpretation of the corresponding heat equation given in [1] we construct a stochastic process X_t such that X_t starting in D moves as the isotropic stable process until the first exit time from D. At the exit time it jumps out of D according to ν. It stays at the exit point y for an exponential time with mean $1/\nu(y,D)$ then jumps back to D and restarts.

We investigate some fundamental properties of X_t, the corresponding semigroup and bilinear form. In particular we prove that the lifetime of X is infinite for $\alpha \in (0,1]$ and finite for $\alpha \in (1,2)$. In the latter case we have $\lim_{t \to \zeta} X_t = 0$, where ζ denotes the lifetime.

We prove that for sufficiently regular functions f the function $u = Gf$ is the solution of the above Neumann problem, where G is the 0-potential of X, i.e., $Gf(x) = E \int_0^\infty f(X_t)\, dt$.

This is a joint work with Krzysztof Bogdan and Damian Fafuła from Wrocław University of Science and Technology.

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LARGE DEVIATIONS FOR STOCHASTIC EVOLUTION EQUATIONS
(PARALLEL TALK)

ESMÉE THEEWIS [1] (JOINT WORK WITH MARK VERAAR [1])

This talk is about a new large deviation result for solutions to quasilinear stochastic evolution equations with small Gaussian noise. We consider a recently developed, general variational framework called the critical variational setting [2]. The Large Deviation Principle (LDP) has been successfully proved for several variational frameworks by means of the so-called weak convergence approach of [3], but often, strong assumptions have to be made on the coefficients in the equation or on the Gelfand triple. In our current large deviation result, we do not require any additional assumption apart from those required for well-posedness of the equation. This leads to numerous applications for which the LDP was not established yet, in particular equations on unbounded domains with gradient noise. The talk is based on [4].

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