FLUCTUATION-THEORY FOR AR(1) PROCESSES
(PLENARY TALK)

GEROLD ALSMEYER

For $\theta > 0$ and i.i.d. real-valued random variables $\xi_1, \xi_2, \ldots$, this talk will consider AR(1) processes of order θ, defined by

$$S_0 = 0 \quad \text{and} \quad S_n = \theta S_{n-1} + \xi_n \text{ for } n \geq 1,$$

and provide some fluctuation-theoretic properties of these processes, which amounts to a look at their behavior as $n \to \infty$, their ladder variables, their global minimum or maximum, and renewal-type properties. Since $(S_n)_{n\geq 0}$ is just an ordinary random walk if $\theta = 1$, the focus will be on the case when $0 < \theta \neq 1$. If time allows the case when θ is replaced with a random variable θ_n in the definition of S_n will also be briefly discussed under the assumption that $(\theta_1, \xi_1), (\theta_2, \xi_2), \ldots$ are i.i.d. taking values in $(0, \infty) \times \mathbb{R}$.

COLLISION TIMES OF MULTIVARIATE BESSEL PROCESSES WITH THEIR WEYL CHAMBER'S BOUNDARY

(PLENARY TALK)

SERGIO ANDRAUS, JAPAN INTERNATIONAL UNIVERSITY
(NICOLE HUFNAGEL, HEINRICH-HEINE-UNIVERSITÄT DÜSSELDORF)

Multivariate Bessel processes, also known as radial Dunkl processes [1, 2, 3] are N-dimensional generalizations of the well-known Bessel processes that depend on the choice of a particular symmetry given by a root system, a set of vectors that remains invariant under reflections generated by its own elements. These processes evolve within a corresponding subset of $\mathbb{R}^N$ called a Weyl chamber, and some particular cases of these processes can be found in mathematical physics, especially those linked with random matrix theory such as the Dyson model [4] and the Wishart-Laguerre processes [5, 6].

One of the more interesting features of these processes is that they are characterized by a series of parameters, called multiplicities, which indicate the strength of the singular repulsive drift which drives them away from the Weyl chamber boundary. Given the root system R, the multiplicities $k(\alpha) > 0,\ \alpha \in R$, and a standard, N-dimensional Brownian motion $\{B(t)\}_{t\geq 0}$, the multivariate Bessel process' evolution, $\{X(t)\}_{t\geq 0}$, is given by

$$\mathrm{d}X(t) = \mathrm{d}B(t) + \sum_{\alpha\in R} \frac{k(\alpha)}{2} \frac{\alpha}{\langle \alpha, X(t)\rangle}\,\mathrm{d}t,\ X(0) = x_0.$$

It turns out that in spite of the singular drift, $X(t)$ hits the Weyl chamber's boundary in finite time almost surely whenever a multiplicity is less than 1/2 [7].

In this talk, we focus on the collision behavior of $X(t)$ after the first hitting time, and making use of the observation that $X(t)$ is self-similar, we realize that the hitting times form a set with fractal features. Our main result is stated as follows.

Theorem 1. *The Hausdorff dimension of collision times with the Weyl chamber boundary is given by*

$$\dim[X^{-1}(\partial W)] = \max\Big\{0, \frac{1}{2} - \min_{\alpha\in R} k(\alpha)\Big\}.$$

We start by outlining a direct proof of this statement for the root system of type A, which corresponds to the Dyson model. For this proof, we make extensive use of asymptotics due to Graczyk and Sawyer [8] which are unknown for other root systems. Then, thanks to a crucial observation by J. Małecki, we show that when all multiplicities are equal,

$$V[X(t)] := \prod_{\alpha\in R} |\langle \alpha, X(t)\rangle|,$$

the squared alternating polynomial of $X(t)$, obeys the SDE of a squared Bessel process with a bi-Lipshitz random time-change for any of the reduced root systems. This allows us to finish our proof by showing that collisions of $X(t)$ with the Weyl chamber boundary have the same Hausdorff dimension as the visits to the origin of a Bessel process. We finish the proof by extending it to arbitrary positive multiplicities via bounds of suitably defined one-dimensional processes [9].

This work is supported by the JSPS Kakenhi grant JP19K14617.

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ON THE SOLUTIONS OF NONLOCAL HAMILTON-JACOBI EQUATIONS WITH GRADIENT NONLINEARITY
(PLENARY TALK)

ANUP BISWAS
INDIAN INST. OF SCIENCE EDUCATION AND RESEARCH
(JOINT WORK WITH ALEXANDER QUAAS AND ERWIN TOPP)

The broad theme of this talk would be to discuss uniqueness of solutions to the nonlocal equation of the form

$$(-\Delta)^s u + H(u, \nabla u) = 0 \quad \text{in } \mathbb{R}^n.$$

This is also known as the Liouville property of the solutions. Similar problems, associated to the Laplace operator, have been studied in great detail but their nonlocal analogue mostly remained unsolved. See the survey article [2] for a detailed discussion on this topic. For $s = 1$ (that is, the Laplacian case), almost all the proofs of Liouville theorem relies on the Bernstein estimate of the gradient. When H is coercive in ∇u and independent of u, a *weak* form of Bernstein estimate can be found in [1]. In this talk, we shall provide a unified approach to prove Liouville type results for a large class of Hamiltonians.

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BRANCHING RANDOM WALKS AND MARTIN BOUNDARIES, PROPERTIES, EXAMPLES AND COUNTEREXAMPLES
(PLENARY TALK)

ELISABETTA CANDELLERO, ROMA TRE UNIVERSITY

Let G be an infinite locally finite and transitive graph. We investigate the relation between supercritical transient branching random walk (BRW) and the Martin boundary of its underlying random walk. We show results regarding the typical (and some atypical) asymptotic directions taken by the particles. We focus on the behavior of BRW inside given subgraphs by putting into relation geometrical properties of the subgraph itself and the behavior of BRW on it. We will also present some examples and counterexamples. (Based on joint works with T.Hutchcroft, D.Bertacchi and F.Zucca.)

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ON TIGHTNESS OF THE MAXIMUM OF BRANCHING BROWNIAN MOTION IN RANDOM ENVIRONMENT AND OF TRANSITION FRONTS TO RANDOMIZED F-KPP EQUATION

(PLENARY TALK)

JIŘÍ ČERNÝ, UNIVERSITY OF BASEL

In the presentation, I will consider two related models: the one-dimensional branching Brownian motion in random environment (BBMRE), and the randomized F-KPP equation (rFKPP). I will discuss the following natural questions:

(1) Given a typical realisation of the environment, are the distributions of the maximal particle of the BBMRE (re-centred around their medians) tight?

(2) For the same environment, is the width of the front of the "travelling-wave" solution to rFKPP uniformly bounded in time?

Surprisingly, it turns out that the answers to these questions can be different. This highlights that—when compared to the settings of homogeneous branching Brownian motion and the F-KPP equation in a homogeneous environment—the introduction of a random environment leads to a much more intricate behaviour.

The presentation is based on joint works with A. Drewitz, L. Schmitz, and P. Oswald.

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ON THE EXPLOSION OF BRANCHING PROCESSES INDEXED BY THE ESSCHER TRANSFORM

(PLENARY TALK)

LOÏC CHAUMONT, UNIVERSITÉ D'ANGERS

Let φ be the branching mechanism of a non conservative continuous state branching process, that is $\int_{0+} d\lambda/|\varphi(\lambda)| < \infty$. We construct, on the same probability space, a family of continuous state branching processes $Z^{(\varepsilon)}$, $\varepsilon \geq 0$ such that for each ε, $Z^{(\varepsilon)}$ has branching mechanism $\varphi^{(\varepsilon)}(\lambda) = \varphi(\lambda+\varepsilon) - \varphi(\varepsilon)$. Then we study the speed of explosion of the family $Z^{(\varepsilon)}$ when $\varepsilon \to 0$. In particular we characterise the functions f with $\lim_{\varepsilon\to 0} f(\varepsilon) = \infty$ and such that the first passage times $\sigma_\varepsilon = \inf\{t : Z_t^{(\varepsilon)} \geq f(\varepsilon)\}$ converge toward the explosion time of $Z^{(0)}$. This is a joint work with my PhD student Clément Lamoureux.

BALANCED MEASURES AND GENERALIZED MARTINGALE TRANSFORMS: SPARSE DOMINATION

(PLENARY TALK)

JOSÉ MANUEL CONDE ALONSO, UNIVERSIDAD AUTÓNOMA DE MADRID

We study higher-complexity Haar shifts, which can be seen as generalizations of martingale transforms, in filtered spaces more general than $\mathbb{R}^d$ with the dyadic system and the Lebesgue measure. In particular, we consider filtrations that can fail to be regular. We study boundedness results for Haar shifts and weighted inequalities through the technique of sparse domination, that we need to modify significantly to fit our context. Some examples of Haar shifts that we can consider are the dyadic Hilbert transform or the dyadic models for the Riesz vector, introduced by Petermichl et al.

This is based on joint work with Jill Pipher and Nathan Wagner (Brown University).

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PROPAGATION PHENOMENON IN SOME INTEGRODIFFERENTIAL EQUATIONS
(PLENARY TALK)

JÉRÔME COVILLE, INRAE, BIOSTATISTIQUE ET PROCESSUS SPATIAUX (BIOSP) AVIGNON - FRANCE
& ICJ UMR5208 - UNIVERSITE CLAUDE BERNARD LYON 1

EMERIC BOUIN, CEREMADE - UNIVERSITÉ PARIS-DAUPHINE, UMR CNRS 7534, PARIS - FRANCE

I will present some recent result on the description of propagation phenomenon in homogeneous environment that are encounter in models described by

$$\partial_t u(t,x) = P.V.\left(\int_{\mathbb{R}} [u(t,x+z) - u(t,x)]\, d\mu(z)\right) + f(u(t,x)) \quad \text{for} \quad t>0, x\in\mathbb{R}$$
$$u(0,x) = u_0(x) \geq 0$$

with $d\mu(z)$ a symmetric Borel measure and $f \in C^1(\mathbb{R})$ such that $f(0) = f(1) = 0$ and $f'(1) < 0$ and being either bistable or an ignition function. In the literature, this equation has been studied for this two class of nonlinearity when the Borel measure $d\mu(z)$ is either bounded and sometimes absolutely continuous with respect to the Lebesgue measure i.e. $d\mu(z) = J(z)\, dz$ with $J \in L^1$ or $d\mu$ is a Lévy measure associated to an α stable process, i.e. $d\mu(z) = J(z)\, dz$ with J having some prescribed behaviour e.g. $J(z) = \frac{1}{|z|^{d+2s}}$. In particular, for this two type of measure, existence of travelling front or acceleration phenomenon have been intensively studied lately. I will present recent generalisations of existence of front solution and acceleration phenomenon in a context of Lévy measures $d\mu$ with no atomic part, i.e. $d\mu$ such that

$$\mu(\{x\}) = 0 \ \text{ for all singleton } \{x\} \qquad \text{and} \qquad \int_{\mathbb{R}} \min\{1, z^2\}\, d\mu(z) < +\infty.$$

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BLOW-UP FOR A GENERAL NONLOCAL EQUATION
(PLENARY TALK)

ARTURO DE PABLO

UNIV. CARLOS III DE MADRID, LEGANÉS, SPAIN

(RAÚL FERREIRA)
(UNIV. COMPLUTENSE DE MADRID, MADRID, SPAIN)

We study the blow-up property for the diffusion equation

$$(\mathcal{D}_t + \mathcal{L}_x)u = u^p, \qquad x \in \mathbb{R}^N,\ 0 < t < T,$$

with $p > 1$, where $\mathcal{D}_t \sim \partial_t^\alpha$, the Caputo fractional derivative in time with $0 < \alpha < 1$, and $\mathcal{L}_x$ is a nonlocal operator in space driven by a Lévy kernel $\mathcal{J}$. We show that the exponent separating the range where all solutions blow up from the range where also global solutions exist, the so-called *Fujita exponent* [3], depends only on the tail of the kernel.

Theorem 1. [2]
i) Assume $\mathcal{J}(z) \le c|z|^{-N-\gamma}$ *for* $|z| > 1$, $\gamma > 0$. *If* $1 < p < 1 + \overline{\gamma}/N$ *then every solution blows up in finite time.*
ii) Assume

$$\mathcal{J}(z) \ge c \begin{cases} |z|^{-N-\omega} & \text{for } |z| > 1, \quad \omega > 0, \\ |z|^{-N-\beta} & \text{for } |z| < 1, \quad \dfrac{N\varpi}{N+\varpi} < \beta < 2. \end{cases}$$

If $p \ge 1 + \varpi/N$ *and either* $\mathcal{D}_t = \partial_t^\alpha$ *or* $N < 2\beta$, *then there exist blowing-up solutions and also global solutions.*
Here

$$\overline{\gamma} = \min\{\gamma, 2\}, \qquad \varpi = \min\{\omega, 2\}.$$

In the stable-like case $\mathcal{L}_x \sim (-\Delta)^{\beta/2}$, the fractional Laplacian with $0 < \beta < 2$, the Fujita exponent is $p_* = 1 + \beta/N$, see also [1] for the exact power case, and [4, 5] for the cases $\alpha = 1$ or $\beta = 2$.

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WEIGHTED T(1) THEOREMS ON SOBOLEV SPACES AND REGULARITY OF BELTRAMI SOLUTIONS
(PLENARY TALK)

FRANCESCO DI PLINIO, UNIVERSITÀ DI NAPOLI "FEDERICO II"

The sharp weighted norm inequality for the Beurling transform, due to Petermichl and Volberg, leads to borderline injectivity of the Beltrami resolvent. When the dilatation coefficient belongs to the Sobolev class on a bounded domain D with suitable boundary regularity, quantitative Sobolev estimates for the Beltrami resolvent are instead related to weighted Sobolev norms of the compression to D of the Beurling transform. These norms are connected to the boundary regularity of D by a testing type theorem for singular integrals on domains. In this talk, we describe a wavelet representation formula [1] and the ensuing general testing type characterization of the weighted Sobolev space boundedness of singular integrals on domains [2], which is sharp and, in this generality, new on Euclidean space as well, recovering the Beurling transform result as a very special case. Applications to Beltrami PDE [3] will also be described. Joint works with W. Green and B. Wick.

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BIASED RANDOM WALKS ON THE SPANNING TREE OF A LADDER GRAPH

(PLENARY TALK)

NINA GANTERT, TECHNICAL UNIVERSITY OF MUNICH
(JOINT WORK WITH ACHIM KLENKE, MAINZ)

We consider a specific random graph which serves as a disordered medium for a particle performing biased random walk. Take a two-sided infinite horizontal ladder and pick a random spanning tree with a certain edge weight c for the (vertical) rungs. Now take a random walk on that spanning tree with a bias $\beta > 1$ to the right. In contrast to other random graphs considered in the literature (random percolation clusters, Galton-Watson trees) this one allows for an explicit analysis based on a decomposition of the graph into independent pieces.

We give an explicit formula for the speed of the biased random walk as a function of both the bias β and the edge weight c. We conclude that the speed is a continuous, unimodal function of β that is positive if and only if $\beta < \beta_c^{(1)}$ for an explicit critical value $\beta_c^{(1)}$ depending on c.

We show that another second order phase transition takes place at another critical value $\beta_c^{(2)} < \beta_c^{(1)}$ that is also explicitly known: For $\beta < \beta_c^{(2)}$ the times the walker spends in traps have second moments and (after subtracting the linear speed) the position fulfills a central limit theorem. We see that $\beta_c^{(2)}$ is smaller than the value of β which achieves the maximal value of the speed. Finally, concerning linear response, we confirm the Einstein relation for the unbiased model ($\beta = 1$) by proving a central limit theorem and computing the variance.

SINGULAR INTEGRALS IN UNIFORMLY CONVEX SPACES
(PLENARY TALK)

TUOMAS HYTÖNEN, AALTO UNIVERSITY

Abstract: We consider the action of finitely truncated singular integral operators on functions taking values in a Banach space. Such operators are bounded for any Banach space, but we show a quantitative improvement over the trivial bound in any space renormalizable with uniformly convex norm, which is equivalent to probabilistic estimates known as martingale type and cotype. The proof, which is based on the representation of a singular integral as an average of dyadic model operators over a random choice of the dyadic decomposition of the domain, follows the broad outline of recent works on similar results for genuinely singular (non-truncated) operators in the narrower class of UMD (unconditional martingale differences) spaces, but our setting, the main theorem, and some aspects of its proof, are new. This is based on arXiv:2310.08926.

WHAT'S A SKEW STABLE LÉVY PROCESS?
(PLENARY TALK)

ALEXANDER IKSANOV (KYIV, UKRAINE)

The skew Brownian motion is a strong Markov process which behaves like a Brownian motion until hitting zero and exhibits an asymmetry at zero. One may wonder what is a natural counterpart of the skew Brownian motion in the situation that an underlying Brownian motion is replaced with a symmetric stable Lévy process with finite mean and infinite variance. Iksanov and Pilipenko in [2] have constructed such a counterpart as a scaling limit of a sequence of perturbed at 0 symmetric α-stable Lévy processes with $\alpha \in (1, 2)$ (continuous-time processes). They called the constructed process a *skew α-stable Lévy process*. In my talk, I'll discuss a simpler construction of the skew α- stable Lévy process as a scaling limit of a sequence of perturbed at 0 standard random walks (random sequences).

The talk is based on the recent joint works [1] and [2] with Congzao Dong (Xi'an, China) and Andrey Pilipenko (Kyiv, Ukraine).

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LOWER BOUNDS OF L^1-NORMS OF NON-HARMONIC TRIGONOMETRIC POLYNOMIALS

(PLENARY TALK)

PHILIPPE JAMING, UNIVERSITÉ DE BORDEAUX, (WITH K. KELLAY & C. SABA)

In this talk, we will present quantitative lower bounds of the L^1 norm of a non-harmonic trigonometric polynomial of the following form:

– let $T > 1$;
– let $(\lambda_j)_{j\geq 0}$ be a sequence of non-negative real numbers with $\lambda_{j+1} - \lambda_j \geq 1$;
– let $(a_j)_{j=0,\ldots,N}$ be a finite sequence of complex numbers. Then

$$C(T)\sum_{j=0}^{N}\frac{|a_j|}{j+1} \leq \frac{1}{T}\int_{-T/2}^{T/2}\left|\sum_{j=0}^{N} a_j e^{2i\pi\lambda_j t}\right| \mathrm{d}t$$

where $C(T)$ is an explicit constant that depends on T only. This provides a quantitative statement of a result by F. Nazarov.

The L^2 analogue is Ingham's Inequality and the harmonic case (λ_j integers) is McGehee, Pigno, Smith's solution of the Littlewood conjecture.

EIGENVALUES, EIGENVECTOR-OVERLAPS, AND PSEUDOSPECTRA OF THE NON-HERMITIAN MATRIX-VALUED STOCHASTIC PROCESSES

(PLENARY TALK)

MAKOTO KATORI, CHUO UNIVERSITY

The non-Hermitian matrix-valued Brownian motion (BM) is the stochastic process of a random matrix whose entries are given by independent complex BMs. We consider a one-parameter extension of this process, which can be regarded as a dynamical version of Girko's elliptic ensemble interpolating the GUE and the Ginibre ensemble of random matrices. Notice that the eigenvalue process of GUE is known as Dyson's BM. For each process in this family, the bi-orthogonality relation is imposed between the left and the right eigenvector processes, which allows for the scale-transformation invariance of the system. There each eigenvalue process is coupled with the eigenvector-overlap process which is a Hermitian matrix-valued process with entries given by products of overlaps of the left and the right eigenvectors. We derive a set of stochastic differential equations and discuss the coupled systems from the viewpoint of pseudospectra, in particular when the initial matrices are nonnormal. The Fuglede–Kadison (FK) determinants of the present matrix-valued processes are regularized by introducing an auxiliary complex variable. Then, we give the stochastic partial differential equations (SPDEs) describing the associated time-dependent random fields in the two-dimensional complex space. Time-dependent point processes (PPs) of eigenvalues and the marked PPs weighted by the diagonal elements of eigenvector-overlap processes are related to the logarithmic derivatives of the regularized FK-determinant random-fields. We also discuss the coupled systems of complex Burgers equations obtained by averaging the SPDEs. The present talk is based on the joint works with Syota Esaki (Fukuoka) and Satoshi Yabuoku (Kitakyushu) [1] and with Saori Morimoto (Chuo) and Tomoyuki Shirai (Kyushu) [2].

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THE ℓ^p NORM OF THE RIESZ–TITCHMARSH TRANSFORM FOR EVEN INTEGER p

(PLENARY TALK)

MATEUSZ KWAŚNICKI, WROCŁAW UNIVERSITY OF SCIENCE AND TECHNOLOGY

Among many discrete variants of the Hilbert transform

$$Hf(x) = \frac{1}{\pi} \operatorname{p.v.}\!\int_{-\infty}^{\infty} \frac{f(x-y)}{y} \, dy,$$

the one introduced by M. Riesz and E.C. Titchmarsh plays a particular role. It is a unitary operator on $\ell^2(\mathbb{Z})$, defined by

$$\mathcal{R}a_n = \frac{1}{\pi} \sum_{k=-\infty}^{\infty} \frac{a_{n-k}}{k+\frac{1}{2}} \, .$$

Riesz and Titchmarsh proved that for $p \in (1, \infty)$, H is a bounded operator on $L^p(\mathbb{R})$, and $\mathcal{R}$ is a bounded operator on ℓ^p. Titchmarsh additionally showed that the corresponding operator norms satisfy

$$\|H\|_p \le \|\mathcal{R}\|_p,$$

and he gave an erroneous proof that in fact equality holds. Almost a century later, the question whether $\|H\|_p = \|\mathcal{R}\|_p$ remains an open problem.

In a recent work [2] with Rodrigo Bañuelos from Purdue University, we give an affirmative answer when p is an even integer (or its Hölder conjugate). Interestingly, our proof requires equality of norms for a closely related discrete Hilbert transform,

$$\mathcal{H}a_n = \frac{1}{\pi} \sum_{k \in \mathbb{Z} \setminus \{0\}} \frac{a_{n-k}}{k} \, ,$$

for *all* $p \ge 2$, not just for even integers. Equality $\|H\|_p = \|\mathcal{H}\|_p$ was proved in [1] using a mixture of probabilistic and analytic tools. Otherwise, our argument is of algebraic nature.

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MAXIMAL DISPLACEMENT OF A TIME-INHOMOGENEOUS $N(T)$-PARTICLES BRANCHING BROWNIAN MOTION
(PLENARY TALK)

PASCAL MAILLARD, UNIVERSITÉ DE TOULOUSE

The N-particles branching Brownian motion (N-BBM) is a branching Markov process which describes the evolution of a population of particles undergoing reproduction and selection. It shares many properties with the N-particles branching random walk (N-BRW), which itself is strongly related to physical p-spin models, or to Derrida's Random Energy Model [5, 6]. The N-BRW can also be seen as the realization of an optimization algorithm over hierarchical data, which is often called *beam search* [3]. More precisely, the maximal displacement of the N-BRW (or N-BBM) can be seen as the output of the beam search algorithm; and the population size N is the "width" of the beam, and (almost) matches the computational complexity of the algorithm.

In this paper, we investigate the maximal displacement at time T of an N-BBM, where $N = N(T)$ is picked arbitrarily depending on T and the diffusion of the process $\sigma(\cdot)$ is inhomogeneous in time. We prove the existence of a *transition* in the second order of the maximal displacement when $\log N(T)$ is of order $T^{1/3}$. When $\log N(T) \ll T^{1/3}$, the maximal displacement behaves according to the Brunet-Derrida correction [4, 2] which has already been studied for N a large constant and for σ constant. When $\log N(T) \gg T^{1/3}$, the output of the algorithm (i.e. the maximal displacement) is subject to two phenomena: on the one hand it begins to grow very slowly (logarithmically) in terms of the complexity N; and on the other hand its dependency in the time-inhomogeneity $\sigma(\cdot)$ becomes more intricate. The transition at $\log N(T) \approx T^{1/3}$ can be interpreted as an "efficiency ceiling" in the output of the beam search algorithm, which extends previous results from [1] regarding an *algorithm hardness threshold* for optimization over the Continuous Random Energy Model.

Based on joint work with Alexandre Legrand [7]

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ASYMPTOTICS FOR LINEAR STUDENT-t LÉVY REGRESSION
(PLENARY TALK)

HIROKI MASUDA, UNIVERSITY OF TOKYO

Objective. Suppose that we have a discrete-time sample $\{(X_{t_j}, Y_{t_j})\}_{j=0}^{[nT_n]}$ with $t_j = j/n$ from the continuous-time (location) regression model

$$Y_t = X_t \cdot \mu + \sigma J_t$$

for $t \in [0, T_n]$, where $X = (X_t)$ is a càdlàg stochastic covariate process in $\mathbb{R}^q$ satisfying some regularity conditions, and $J = (J_t)$ is a Lévy process such that the distribution of J_1 has the (scaled) Student-t density $f(x;\nu) = \dfrac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi}\Gamma(\frac{\nu}{2})}(1+x^2)^{-(\nu+1)/2}$. Our statistical model is indexed by the unknown parameter $\theta := (\mu, \sigma, \nu) \in \mathbb{R}^q \times (0,\infty) \times (0,\infty)$. We want to estimate the true value $\theta_0 = (\mu_0, \sigma_0, \nu_0)$ of θ when $T_n \to \infty$ for $n \to \infty$: how to construct an easy-to-compute and well-behaving estimator $\hat{\theta}_n = (\hat{\mu}_n, \hat{\sigma}_n, \hat{\nu}_n)$?

The talk is based on a joint work with Lorenzo Mercuri (University of Milan, Italy) and Yuma Uehara (Kansai University, Japan). The full details are available in the preprint "Quasi-likelihood analysis for Student-Lévy regression" available at arXiv:2306.16790.

Brief summary of the statements. Although the conditional likelihood given X can be written through a Fourier inversion formula, it is rather intractable. We have the bottleneck that the Student-t distribution is not closed under convolution, making it difficult to estimate all the parameters based on the high-frequency time scale. To efficiently deal with that intricate nature from both theoretical and computational points of view, we propose the fully explicit two-step procedure which is essentially based on the fact that J is locally Cauchy distributed ($h^{-1}J_h \xrightarrow{\mathcal{L}}$ Cauchy as $h \to 0$ in an L^1-local sense):

- First, we make use of the *Cauchy quasi-likelihood* for estimating (μ, σ);
- Second, we construct the *Student-t quasi-likelihood* based on $f(x;\nu)$ with the unit-period residual sequence to estimate the remaining degrees of freedom.

Under regularity conditions, we will obtain *asymptotic normality* and *uniform integrability* of the scaled estimators $\sqrt{nB_n}(\hat{\mu}_n - \mu_0, \hat{\sigma}_n - \sigma_0)$ and $\sqrt{T_n}(\hat{\nu}_n - \nu_0)$, where the sequence $B_n \le T_n$ satisfies a certain condition; B_n represents the percentage of the data volume used in the first step. In particular, we will quantitatively reveal that using full data in the first step may destroy the asymptotic normality because of the accumulation of the small-time local Cauchy approximation error, showing the need for data thinning; in particular, we will need $B_n/T_n \to 0$ to derive the clean-cut joint asymptotic normality of the proposed estimators. It is also possible to deduce the joint asymptotic normality of the quasi-likelihood estimators.

Moreover, we can modify the main result to handle a sample $(Y_{t_j})_{j=0}^{[nT_n]}$ from an ergodic solution to the Markov process ($Y_0 = 0$) described by $Y_t = \mu \cdot \displaystyle\int_0^t b(Y_s)ds + \sigma J_t$, where $\nu_0 > 2$ and $b : \mathbb{R} \to \mathbb{R}^q$ is a known measurable function; the corresponding X in this case is the Riemann approximation of $\int_0^\cdot b(Y_s)ds$. Also presented is a possible extension to a class of locally stable regression models based on a non-Gaussian stable quasi-likelihood.

FRACTIONAL LAPLACIANS, BOUNDARY CONDITIONS, AND SELF-ADJOINTNESS
(PLENARY TALK)

DELIO MUGNOLO, FERNUNIVERSITÄT IN HAGEN

We consider the censored fractional Laplacian which, under appropriate conditions, is symmetric but fails to be self-adjoint. We offer a gentle introduction to the theory of boundary triples and use it to parametrize all self-adjoint and all Markovian extensions of this operator in one dimension. As a by-product, two natural realizations of the fractional Laplacian on metric graphs can also be introduced.

This is joint work with Jussi Behrndt and Markus Holzmann (TU Graz).

LIMIT THEOREMS FOR OCCUPATION TIMES OF SYMMETRIC MARKOV PROCESSES

(PLENARY TALK)

JUAN CARLOS PARDO (CIMAT, MX) (JOSEPH NAJNUDEL, UNIVERSITY OF BRISTOL, AND JU-YI YEN, UNIVERSITY OF CINCINNATI)

Limit theorems of occupation times for Markov process has attracted a lot of interest since the seminal work of Darling an Kac [1] in the late fifties of the last century. In [1], the authors generalised and unified previous results by showing that, under suitable conditions, such limit distribution must be Mittag-Leffler. The approach used in [1] relies on an analytic tools which allow them to apply the celebrated Tauberian theorem. In this talk, we provide a probabilistic approach to such limit theorems but for strongly symmetric Markov process with values in an abstract space and with finite potential densities. Our approach provides further information of the appearance of such limiting distributions. Our methodology relies on excursion theory and on a generalisation of the second Ray-Knigth theorem of such families of processes due to Eisenbaum et al [2].

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BRANCHING PROCESSES, COALESCENTS, AND MOMENT DUALITY
(PLENARY TALK)

JOSÉ LUIS PÉREZ (CIMAT).

In this talk, we will review some recent results that provide the notion of a generalized ancestry for continuous-state branching processes by means of moment duality. This generalization includes cases such as the duality between pairwise branching and efficiency, and in the symmetric case, a relationship between branching processes and coalescents is uncovered. Finally, we will study the genealogy of asymmetric populations by introducing the partial order of adaptation and the asymmetric ancestral selection graph.

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BREGMAN VARIATION OF SEMIMARTINGALES AND ITS APPLICATIONS
(PLENARY TALK)

KATARZYNA PIETRUSKA-PAŁUBA, UNIVERSITY OF WARSAW

The talk will be devoted to the notion of Bregman variation of semimartingales. This is a generalization of quadratic variation, based on a convex function $\phi(t)$ rather than the usual quadratic function $\phi(t) = t^2$. We shall present the construction, some of its properties, and then applications: to several versions of Hardy-Stein identities (some of them new) and Bunkholder's inequality for martingales.

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NORMAL AND POISSON APPROXIMATION OF SUBGRAPH COUNTS WITH FIXED ENDPOINTS IN THE RANDOM-CONNECTION MODEL

(PLENARY TALK)

NICOLAS PRIVAULT, SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES, NANYANG TECHNOLOGICAL UNIVERSITY, JOINT WORK WITH QINGWEI LIU

We consider the count of subgraphs with an arbitrary configuration of $m \geq 0$ endpoints in the random-connection model based on a Poisson point process on $\mathbb{R}^d$. We present combinatorial expressions for the computation of the cumulants and moments of all orders of such subgraph counts, which allow us to estimate the growth of cumulants as the intensity of the underlying Poisson point process goes to infinity. As a consequence, we obtain a central limit theorem with explicit convergence rates under the Kolmogorov distance, and connectivity bounds. When connection probabilities are of order $\lambda^{-\alpha}$, we derive a threshold $\alpha_* > 0$ such that normal approximation for subgraph counts holds when $\alpha \in (0, \alpha_*)$, and a Poisson limit result holds if $\alpha = \alpha_*$.

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IMPROVED ASYMPTOTIC ESTIMATES FOR THE HEAT EQUATION IN EXTERIOR DOMAINS

(PLENARY TALK)

FERNANDO QUIRÓS, U. AUTÓNOMA DE MADRID & ICMAT

Giving good lower bounds for the Dirichlet heat kernel in the complement of a compact set is surprisingly a relatively new result [1, 2]. We use entropy methods and some recent advances in logarithmic Sobolev inequalities to improve the available results, obtaining optimal lower and upper asymptotic bounds for large times with an explicit approach rate. Asymptotic results are also given for solutions of the heat equation in exterior domains with integrable initial data having suitable finite moments.

This is a work in collaboration with José A. Cañizo and Alejandro Gárriz.

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WEAK WEIGHTED ESTIMATES FOR THE SQUARE FUNCTION. THE CRITICAL CASE.

(PLENARY TALK)

MARIA CARMEN REGUERA, UNIVERSITY OF MÁLAGA

Perhaps one of the last problems to be understood in the area of weighted sharp estimates is that of the weak weighted estimates for the square function at the critical point $p = 2$. We will discuss the conjecture and report on recent progress on a relevant problem in the matrix weight setting. This is joint work with Sandra Pott and Gianmarco Brocchi.

REPRESENTATIONS OF THE WIENER-HOPF FACTORIZATION FOR MARKOV ADDITIVE PROCESSES

(PLENARY TALK)

RIVERO, VÍCTOR, CENTRO DE INVESTIGACIÓN EN MATEMÁTICAS, GUANAJUATO, MÉXICO & DEPARTMENT OF STATISTICS, UNIVERSITY OF WARWICK, UK.

A key stone in the fluctuation theory of Levy processes is the Wiener-Hopf factorization of the characteristic exponent, and most of the research in this topic involves consequences or facts about it. It contains the suitable information to describe the so-called upward and ladder height process associated to a Lévy process. These processes lead to a deep understanding of the running supremum and infimum of the path, and in principle, have a tractable structure as their paths are non-decreasing. Recently there has been a renewed interest in the class of Markov additive processes, due in particular to its connection to self-similar Markov processes and to the richness of its structure. These processes can be thought as Lévy processes modulated by an auxiliary Markov process, as they are composed of an additive and a driving Markovian component. The additive component bears various nice properties of Levy processes, as for instance stationarity and homogeneity of their increments, but conditionally to the driving component. When the driving component is a Markov chain with a finite state space, we can think of the driving part as a concatenation of Levy processes, and hence we get easily convinced that many results for Levy processes should have a translation into this setting. In particular, it has been proved that the Wiener-Hopf factorization holds true, and that several of its representation can be extended to this setting. A particularly interesting one was obtained by Vigon, it relates the Levy measures of the Levy process and those of their upward and downward ladder height processes, the so-called equations amicales. This has been recently proved to hold in the Markov additive context by Döering, Trottner and Watson, under the assumption of the state space of the driving process being finite. In this talk I will describe the form the Wiener-Hopf factorization takes in the more general setting where the driving process has a general state space, and prove that the equations amicales hold also true in this general setting. The proof we will provide of this result uses some general principles of Markov processes, as for instance duality, resolvents and infinitesimal generators. This talk is based in a work in progress in collaboration with Andreas Kyprianou and Mehar Motala.

FRACTIONAL CONVEXITY
(PLENARY TALK)

JULIO DANIEL ROSSI,
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We introduce a notion of fractional convexity that extends naturally the usual notion of convexity in the Euclidean space to a fractional setting. With this notion of fractional convexity, we study the fractional convex envelope inside a domain of an exterior datum (the largest possible fractional convex function inside the domain that is below the datum outside) and show that the fractional convex envelope is characterized as a viscosity solution to a non-local equation that is given by the infimum among all possible directions of the 1−dimensional fractional Laplacian. For this equation we prove existence, uniqueness and a comparison principle (in the framework of viscosity solutions). In addition, we study the behavior of the fractional convexity when the fractional parameter goes to 1 and prove that the fractional convex envelope inside a strictly convex domain of a continuous and bounded exterior datum converges when $s \nearrow 1$ to the classical convex envelope of the restriction to the boundary of the exterior datum.

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ON POISSON EQUATION WITH MEASURE DATA
(PLENARY TALK)

ANDRZEJ ROZKOSZ, NICOLAUS COPERNICUS UNIVERSITY IN TORUŃ
(JOINT WORK WITH TOMASZ KLIMSIAK)

Let D be a bounded domain in $\mathbb{R}^d$, $d \geq 2$. We consider the Dirichlet problem

$$-Lu = \mu \quad \text{in } D, \quad u|_{\partial D} = 0,$$

where L is the operator operator associated with a symmetric transient regular Dirichlet form and μ is a (signed) Borel measure on D having finite total variation. It is known that μ admits a (unique) decomposition

$$\mu = \mu_d + \mu_c, \quad \text{where} \quad \mu_d \ll \text{Cap}, \quad \mu_d \bot \mu_c,$$

and Cap is the capacity determined by L (μ_d is the so-called diffuse or smooth part of μ and μ_c is the concentrated part).

We introduce new function space which allows us to distinguish between solutions with diffuse measures (i.e. $\mu_c = 0$) and solutions with general Borel measures, i.e. measures with nontrivial concentrated part. This space can by characterized analytically in terms of the Poisson kernel associated with L or probabilistically by using the notion of Doob class (D) of processes naturally associated with L.

In the second part we will present a reconstruction formula. It relates the behaviour of the energy of solution u on the set where it is very large to the concentrated part μ_c of μ. In case $L = \Delta$ it has the form

$$\lim_{n\to\infty} \frac{1}{n} \int_{\{n \leq u \leq 2n\}} \eta \nabla u \cdot \nabla u \, dx = \int_D \eta \, d\mu_c^+, \quad \eta \in C_c(D),$$

(see [1]). We generalize the above formula to the case with general operator L. My talk is based on [2].

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THE LIOUVILLE VS. THE UNIQUE CONTINUATION PROPERTIES FOR FOURIER MULTIPLIER OPERATORS WHICH GENERATE STOCHASTIC PROCESSES
(PLENARY TALK)

RENÉ L. SCHILLING, TU DRESDEN (JOINT WITH DAVID BERGER, TU DRESDEN)

We discuss necessary and sufficient criteria for certain Fourier multiplication operators to satisfy the Liouville property (bounded harmonic functions are a.s. constant) and the local continuation property (bounded functions, that are harmonic and identically zero on a domain, are a.s. zero on the whole space). Since the operators generate stochastic processes, there is also a probabilistic interpretation of these findings.

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ON THE CONTACT PROCESS ON DYNAMICAL RANDOM GRAPHS WITH DEGREE DEPENDENT DYNAMICS
(PLENARY TALK)

ANJA STURM, UNIVERSITY OF GÖTTINGEN
(NATALIA CARDONA-TOBON, MARCEL ORTGIESE AND MARCO SEILER)

Recently, there has been increasing interest in interacting particle systems on evolving random graphs, respectively in time evolving random environments. In this talk we present results on the contact process in an evolving edge random environment on infinite (random) graphs. The classical contact process models the spread of an infection in a structured population. The structure is given by a graph and the infection is passed on along the edges with rate λ while recovery from the infection happens spontaneously with rate 1. In an edge random environment the edges of the underlying (random) graph may be be dynamically opened and closed to infection.

We first give an overview over recent results. Then, we in particular consider (infinite) Bienaymé-Galton-Watson (BGW) trees as the underlying random graph. Here, we focus on an edge random environment that is given by a dynamical percolation whose opening and closing rates and probabilities are degree dependent. This means that any edges between two vertices x and y with degrees d_x and d_y is independently updated with rate $v(d_x, d_y)$ and subsequently again declared open (or otherwise closed) with probability $p(d_x, d_y)$. Our results concern the impact of v and p on the critical infection rate for weak (global) and strong (local) survival of the infection. Specifically, we establish conditions under which the contact process undergoes a phase transition.

For a general connected locally finite graph we provide sufficient conditions for the critical infection rate to be strictly positive. Furthermore, in the setting of BGW trees, we provided conditions on the offspring distribution as well as on v and p so that the process survives strongly with positive probability for all positive values of the infection rate. In particular, if the offspring distribution follows a power law (or has a stretched exponential tail) and the connection probability is given by a product kernel, i.e., $p(d_x, d_y) = (d_x d_y)^{-\alpha}$ for a positive α, (or a maximum kernel) and the update speed exhibits polynomial behaviour, we provide a quite complete characterisation of the phase transition.

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THE BROWNIAN WEB AND NET
(PLENARY TALK)

JAN M. SWART, CZECH ACAD. SCI., INST. INFORM. TH. & AUTOM.

The Brownian web can informally be described as coalescing one-dimensional Brownian motions, started from every point in space and time. Despite its apparent simplicity it has turned out to be a flexible tool in the study of a number of other one-dimensional stochastic continuum models. It was first studied by Arratia who was interested in scaling limits of the voter model [1]. It is related to the infinite noise limit of the heat equation with Wright-Fisher white noise [12]. Tóth and Werner studied the Brownian web in order to understand the scaling limit of a self-repelling random walk [13]. Fontes, Isopi, Newman, and Ravishankar first gave the Brownian web its name and viewed is as a random compact set of paths taking values in a suitable Polish space [4]. Sun and Swart [11] used it to construct the Brownian net that is the scaling limit of coalescing random walks that additionally branch with a small probability. Newman, Ravishankar, and Schertzer gave an alternative construction of the Brownian net [7], extended it by adding killing [8], and used it to study the scaling limits of small voter model perturbations [9]. In the meantime, Schertzer, Sun, and Swart [10] used the Brownian web and net to study a class of stochastic flows of kernels introduced by Le Jan and Raimond [6] and Howitt and Warren [5]. The Brownian web has been shown to be the scaling limit of various drainage models such as the two-dimensional directed spanning forest [3]. Most recently, Cannizzaro and Hairer [2] used the Brownian web to construct a randomly evolving interface that they called the Brownian castle.

In my talk, I will give a gentle introduction to the theory of the Brownian web and net and will try to show at least some of their applications.

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COLLISIONS OF THE SUPERCRITICAL KELLER-SEGEL PARTICLE SYSTEM

(PLENARY TALK)

YOAN TARDY, CMAP (POLYTECHNIQUE,X) (WITH NICOLAS FOURNIER, LPSM)

We study a particle system naturally associated to the 2-dimensional Keller-Segel equation. It consists of N Brownian particles in the plane, interacting through a binary attraction in $\theta/(Nr)$, where r stands for the distance between two particles. When the intensity θ of this attraction is greater than 2, this particle system explodes in finite time. We assume that $N > 3\theta$ and study in details what happens near explosion. There are two slightly different scenarios, depending on the values of N and θ, here is one: at explosion, a cluster consisting of precisely k_0 particles emerges, for some deterministic $k_0 \geq 7$ depending on N and θ. Just before explosion, there are infinitely many (k_0-1)-ary collisions. There are also infinitely many (k_0-2)-ary collisions before each (k_0-1)-ary collision. And there are infinitely many binary collisions before each (k_0-2)-ary collision. Finally, collisions of subsets of $3, \ldots, k_0-3$ particles never occur. The other scenario is similar except that there are no (k_0-2)-ary collisions.

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DEGREE-PENALIZED CONTACT PROCESSES

(PLENARY TALK)

DANIEL VALESIN (UNIVERSITY OF WARWICK)
JOINT WORK WITH J. KOMJÁTHY AND Z. BARTHA

The contact process is a model for the spread of an infection in a graph. Vertices can be either healthy or infected; infected vertices recover with rate 1 and send the infection to each neighbor with rate λ. A key question of interest is: if we start the process with a single infected vertex, can the infection survive forever with positive probability? This typically depends on the graph and on the value of λ; for instance, on integer lattices, there is a critical value of λ at which the survival probability changes from zero to strictly positive. However, on graphs that include vertices of high degree, such as Galton–Watson trees with heavy-tailed offspring distributions, it has been observed that the infection survives with positive probability for all values of λ, no matter how small. This is because high-degree vertices sustain the infection for a long time and send the infection to each other. In this work, we investigate this survival-for-all-λ phenomenon for a modification of the contact process, which we introduce and call the penalized contact process. In this new process, vertex u transmits the infection to neighboring vertex v with rate $\lambda / \max(\text{degree}(u), \text{degree}(v))^{\mu}$, where $\mu > 0$ is an additional parameter (called the penalization exponent). This is inspired by considerations from social network science: people with many contacts do not have the time to infect their neighbors at the same rate as people with fewer contacts. We show that the introduction of this penalty factor introduces a rich range of behavior for the phase diagram of the contact process on Galton–Watson trees. We also show corresponding results for the penalized contact process on finite graphs obtained from the configuration model, which locally converge to Galton–Watson trees.

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FREEZING LIMITS OF CALOGERO-MOSER-SUTHERLAND PARTICLE PROCESSES
(PLENARY TALK)

MICHAEL VOIT, FAKULTÄT MATHEMATIK, TECHNISCHE UNIVERSITÄT DORTMUND, GERMANY

One-dimensional Calogero-Moser-Sutherland particle models with N particles can be regarded as diffusions on suitable subsets of $\mathbb{R}^N$ like Weyl chambers and alcoves with second order differential operators as generators, which are singular on the boundaries of the state spaces. The most relevant examples are multivariate Bessel processes and Heckman-Opdam processes which are related to special functions associated with root systems. These models include Dyson's Brownian motions, multivariate Jacobi processes and, for fixed time, β-Hermite, β- Laguerre, and β-Jacobi, i.e., MANOVA ensembles. In some cases, they are related to Brownian motions on the classical symmetric spaces.

The processes depend on parameters which have the interpretation of an inverse temperature. We review several freezing limits for fixed N when one or several parameters tend to ∞. In many cases, the limits are normal distributions and, in the process case, Gaussian processes where the parameters of the limit distributions are described in terms of solutions of ordinary differential equations which appear as frozen versions of the particle diffusions. We also discuss connections of these ODES with the zeros of the classical orthogonal polynomials and polynomial solutions of some one-dimensional inverse heat equations.

The talk is partially based on joint work with Sergio Andraus, Martin Auer, Kilian Herrmann, Margit Rösler, and Jeannette Woerner.

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MARKOV PROCESSES WITH JUMP KERNELS DECAYING AT THE BOUNDARY

(PLENARY TALK)

ZORAN VONDRAČEK, UNIVERSITY OF ZAGREB

In this talk I will explain some aspects of a general theory for non-local singular operators of the type

$$L_\alpha^{\mathcal{B}} f(x) = \lim_{\epsilon\to 0}\int_{D,\,|y-x|>\epsilon} \big(f(y)-f(x)\big)\mathcal{B}(x,y)|x-y|^{-d-\alpha}\,dy,$$

and

$$Lf(x) = L_\alpha^{\mathcal{B}} f(x) - \kappa(x)f(x),$$

in case D is a $C^{1,1}$ open set in $\mathbb{R}^d$, $d \geq 2$. The function $\mathcal{B}(x,y)$ above may vanish at the boundary of D, and the killing potential κ may be subcritical or critical.

From a probabilistic point of view we study the reflected process on the closure $\overline{D}$ with infinitesimal generator $L_\alpha^{\mathcal{B}}$, and its part process on D obtained by either killing at the boundary ∂D, or by killing via the killing potential $\kappa(x)$. The general theory developed in this work (i) contains subordinate killed stable processes in $C^{1,1}$ open sets as a special case, (ii) covers the case when $\mathcal{B}(x,y)$ is bounded between two positive constants and is well approximated by certain Hölder continuous functions, and (iii) extends the main results known for the half-space in $\mathbb{R}^d$. The main results are the boundary Harnack principle and its possible failure, and sharp two-sided Green function estimates. The results on the boundary Harnack principle completely cover the corresponding earlier results in the case of half-space. The Green function estimates extend the corresponding earlier estimates in the case of half-space to bounded $C^{1,1}$ open sets.

Joint work with Soobin Cho, Panki Kim and Renming Song.

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