ESTIMATES OF KERNELS FOR SCHRÖDINGER SEMIGROUPS

(POSTER)

MIŁOSZ BARANIEWICZ

We consider the Schrödinger operator of the form $H = -\Delta + V$ acting in $L^2(\mathbb{R}^d, dx)$, $d \geq 1$, where the potential $V : \mathbb{R}^d \to [0, \infty)$ is a locally bounded function. The corresponding Schrödinger semigroup $\left\{e^{-tH} : t \geq 0\right\}$ consists of integral operators, i.e.

$$e^{-tH} f(x) = \int_{\mathbb{R}^d} u_t(x, y) f(y) dy, \quad f \in L^2(\mathbb{R}^d, dx), \ t > 0. \tag{1}$$

I will present new estimates for heat kernel of $u_t(x, y)$. Our results show the contribution of the potential is described separately for each spatial variable, and the interplay between the spatial variables is seen only through the Gaussian kernel.

I will present applications of those theorems for two common classes of potentials. For confining potentials we get two sided estimates and for decaying potentials we get new upper estimate.

The poster is based on joint work with Kamil Kaleta [1].

References

[1] M. Baraniewicz, K. Kaleta, *Integral kernels of Schrödinger semigroups with nonnegative locally bounded potentials.* To appear in Studia Mathematica (ArXiv:2302.13886), 2023+.

ASYMPTOTICS OF DENSITIES FOR LÉVY PROCESSES WITH STOCHASTIC RESETTING

(POSTER)

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Dive into the fascinating realm of Lévy processes intertwined with the intriguing concept of stochastic resetting, as we embark on a journey to unveil the hidden patterns governing their asymptotic behavior. We will define the concept of resetting and present the pleasant formula it takes for Levy processes. We derive formula for its density and find asymptotic behavior, as space and time tends to infinity.
Besides proving the behavior for process, obtained from processes such as, for example stable subordinators or unimodal and isotropic process, we will focus on demonstrating the behavior of the non-trivial case like the relativistic stable process. In this case, the asymptotics of the density change depending on both: the ratio of the parameters of the mass of the non-reset process and the resetting intensity and on the speed at which time and space escape to infinity. For this case we proved that

Theorem 1. *Let p_λ be density of process $X \in \mathbb{R}^d$, obtained from α-stable relativistic process with mass m. If reset occurred at poisson arrival times with intensity λ then*

$$p_\lambda(t,x) \sim \begin{cases} C_1 |x|^{\frac{1-d-\alpha}{2}} e^{-m^{1/\alpha}|x|}, & m < \lambda \\ C_2 |x|^{\frac{\alpha-1-d}{2}} e^{-m^{1/\alpha}|x|} \int_0^{\frac{t}{|x|^{\alpha/2}}\sqrt{2^\alpha m}} \eta(y,1)dy, & m = \lambda \\ C_3 |x|^{\frac{\alpha-1-d}{2}} e^{-m^{1/\alpha}|x|} \int_0^{\frac{t}{|x|^{\alpha/2}}\sqrt{2^\alpha m}} e^{(m-\lambda)\frac{|x|^{\alpha/2}}{\sqrt{2^\alpha m}} y} \eta(y,1)dy, & m > \lambda \quad \frac{t}{|x|^{\alpha/2}} < M \end{cases}$$

where η is density of $\alpha/2$-stable subordinator, M arbitrary constant and C_1, C_2, C_3 are known constant dependent only on m, λ, d and α.

The case where $\frac{t}{|x|^{\alpha/2}} > M$ and $m > \lambda$, is being investigated.

SUBGRAPH COUNT IN NON-UNIFORM RANDOM HYPERGRAPHS
(POSTER)

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We focus on the problem of counting small subhypergraphs in a non-uniform random hypergraph. The model of random hypergraph $\mathcal{H}(n, (p_r))$ that we consider, generalizes binomial Erdős-Rényi model $\mathcal{G}(n, p)$ in a natural way. Informally, given a set of n vertices every nonempty subset can independently form a hyperedge, and probability p_r of its presence is associated with the size of that hyperedge. If restricted to hyperedges of the same size r, the hypergraph is called r-uniform. Hence, a 2-uniform hypergraph is a simple graph.

Let N_n^H be the number of isomorphic copies of a given hypergraph H in a random hypergraph $\mathcal{H}(n, (p_r)), p_r = p_r(n)$. We derive a necessary and sufficient asymptotic normality condition for the rescaled random variable N_n^H. Moreover, upper bounds of convergence rate are also discussed.

Most of the literature on random hypergraphs is devoted to the uniform case, but our new results cover the general non-uniforn case. The obtained results are compared with the previous ones for uniform random hypergraphs [1] and binomial random graphs [2].

References

[1] Peter de Jong, A central limit theorem with applications to random hypergraphs. *Random Structures & Algorithms* 8.2 (1996), s. 105–120.

[2] A. Ruciński, When are small subgraphs of a random graph normally distributed?, *Probab. Theory Related Fields* 78 (1988), no. 1, 1–10.

DISCRETE FEYNMAN-KAC SEMIGROUPS: RELATIONS BETWEEN ERGODICITY AND ULTRACONTRACTIVITY IN ℓ^p SPACES (POSTER)

MATEUSZ ŚLIWIŃSKI, POLITECHNIKA WROCŁAWSKA (JOINT WORK WITH WOJCIECH CYGAN, KAMIL KALETA AND RENÉ SCHILLING)

We present results of our investigation of a particular discrete-time counterpart of the Feynman–Kac semigroup with a confining potential in a countably infinite space. These findings are a continuation of our work, described in detail in the paper [1]. We focus on Markov chains with the direct step property, which is satisfied by a wide range of typically considered kernels. In our joint work with Wojciech Cygan, René Schilling and Kamil Kaleta [3], we introduce the concept of progressive intrinsic ultracontractivity (pIUC) and investigate links between pIUC of Feynman–Kac semigroups, their uniform quasi-ergodicity and uniform ergodicity of their intrinsic semigroups. We conclude with a simple observation which allows us to linearly interpolate certain related operators on ℓ^p spaces, $1 \leqslant p \leqslant \infty$.

References

[1] W. Cygan, K. Kaleta, M. Śliwiński, *Decay of harmonic functions for discrete time Feynman– Kac operators with confining potentials*, ALEA, Lat. Am. J. Probab. Math. Stat. 19, 1071–1101 (2022).

[2] Kamil Kaleta, René Schilling, *Progressive intrinsic ultracontractivity and heat kernel estimates for non-local Schrödinger operators*, Journal of Functional Analysis, volume 279, Issue 6

[3] W. Cygan, K. Kaleta, R. Schilling, M. Śliwiński, *Kernel estimates for discrete Feynman–Kac operators*, preprint, 2023.

INTRINSIC ULTRACONTRACTIVITY OF FEYNMAN-KAC SEMIGROUPS FOR CYLINDRICAL STABLE PROCESSES
(POSTER)

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The following nonlocal, anisotropic, singular Lévy operator in $\mathbb{R}^{3N}$

$$L^{(rel)}f(x) = -\sum_{k=1}^{N}\sqrt{-\frac{\partial^2}{\partial x_{3k-2}^2} - \frac{\partial^2}{\partial x_{3k-1}^2} - \frac{\partial^2}{\partial x_{3k}^2} + m^2} + Nm,$$

where $m > 0$, $N \in \mathbb{N}$, plays an important role in some models of relativistic quantum mechanics. For example, in Lieb and Seiringer's book [1] it is a kinetic term in relativistic Schrödinger Hamiltonian for N electrons.

In this poster, we consider the Schrödinger operator

$$K = K_0 + V,$$

where

$$K_0 = \sqrt{-\frac{\partial^2}{\partial x_1^2}} + \sqrt{-\frac{\partial^2}{\partial x_2^2}}.$$

This is an example of a nonlocal, anisotropic, singular Lévy operator and may be viewed as a simplified version of $L^{(rel)}$. We study potentials $V : \mathbb{R}^2 \to \mathbb{R}$ such that $V(x)$ goes to infinity as $|x| \to \infty$. The operator $-K_0$ is a generator of a process $X_t = (X_t^{(1)}, X_t^{(2)})$ such that $X_1^{(1)}$, $X_2^{(2)}$ are independent symmetric Cauchy processes in $\mathbb{R}$.

We define the Feynman-Kac semigroup

$$T_t f(x) = E^x\left(\exp\left(-\int_0^t V(X_s)\,ds\right) f(X_t)\right).$$

Operators T_t are compact for every $t > 0$. There exists an orthonormal basis $\{\phi_n\}_{n=1}^{\infty}$ in $L^2(\mathbb{R}^2)$ and a sequence of eigenvalues $\{\lambda_n\}_{n=1}^{\infty}$, $0 < \lambda_1 \le \lambda_2 \le \lambda_3 \le \dots$, $\lim_{n\to\infty}\lambda_n = \infty$ such that $T_t\phi_n = e^{-\lambda_n t}\phi_n$. We can assume that ϕ_1 is positive and continuous on $\mathbb{R}^2$. The main result concerns estimates for ϕ_1 and intrinsic ultracontractivity of the semigroup T_t under certain conditions on the potential V.

This poster is based on my master's thesis [2].

References

[1] E. Lieb, R. Seiringer, *The stability of matter in quantum mechanics*, Cambridge University Press, Cambridge, 2010.

[2] K. Sztonyk, *Półgrupy Feynmana-Kaca dla procesów stabilnych o niezależnych składowych*, Master's thesis, 2023.